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Clothesline Math

The Master **Number Sense** Maker

10   20   50

-1   -x   0   x $\dfrac{1}{2x}$

0   $\dfrac{1}{4}$ $\dfrac{1}{3}$ $\dfrac{1}{2}$ 1
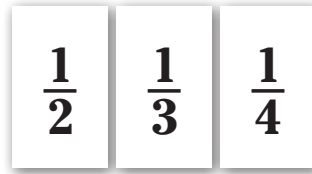
Chris Shore

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A Universal Example

$$\boxed{\dfrac{1}{2}} \quad \boxed{\dfrac{1}{3}} \quad \boxed{\dfrac{1}{4}}$$

Objective: Accurately place and space three unit fractions on an open number line.

Teacher: Good day, class. Today, I am introducing you to the clothesline. This number line will help us better understand the math we are studying. Today, though, we will start with a topic we already know in order to show you how the Clothesline works.

First, we call it the Clothesline because it is literally a clothesline with clothespins that we will use to show that certain values are equal.

I will give you numbers written on folded cards that look like tents. This shape makes it really easy to hang our numbers on the clothesline.

The teacher shows students the folded cards, as shown in Figure I.1.

Figure I.1 Folded Clothesline Cards
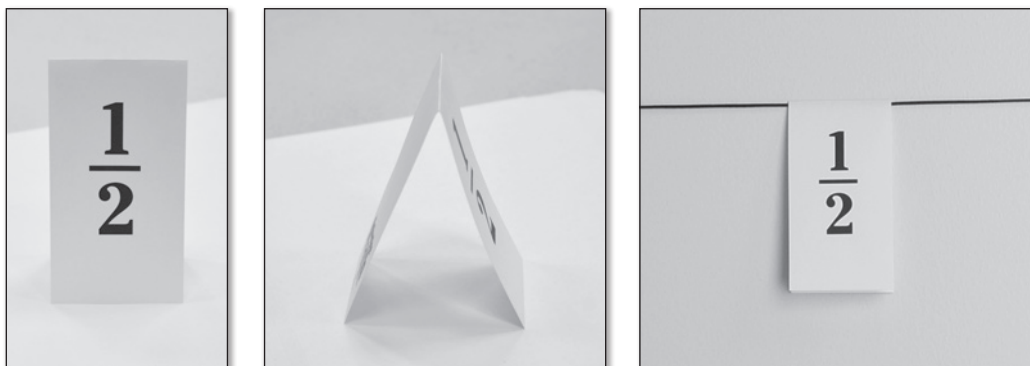
You will notice there are no "benchmarks"—those numbers and tick marks you usually see on a number line. That's because this is an open number line, so you get to choose how to label it. When it is your turn to place numbers on the number line, I have benchmarks printed and ready for you on the table at the front of the room.

The teacher gestures to the pile of benchmark cards at the front of the room, as shown in Figure I.2.

Figure I.2 Sample Benchmark Cards
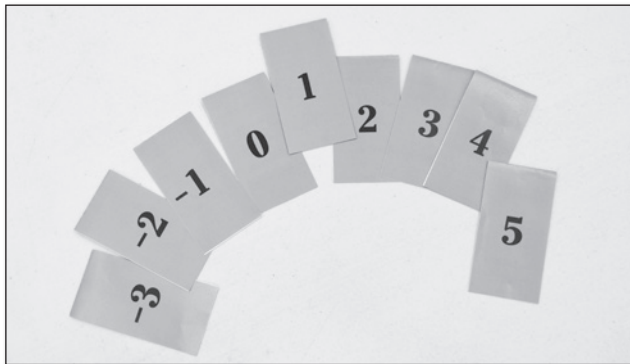
> If you want to use a benchmark that I don't have up here, I have provided blank cards that you may use to create your own.

The teacher shows blank cards sitting next to the benchmark cards, as shown in Figure I.3.

Figure I.3 Blank Cards
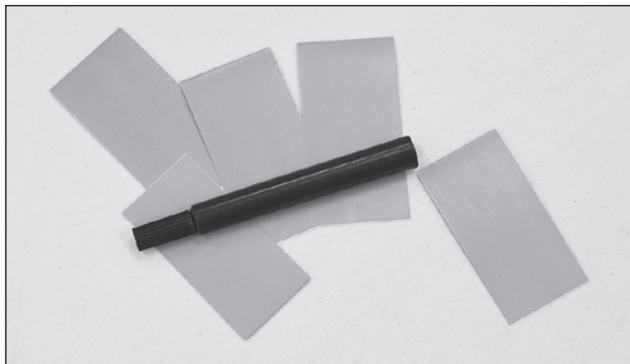
> I'm going to give this first set of numbers to one group that will come to the front of the class and place them on the clothesline. While that group is up here, the rest of you will draw your responses on your lapboards with your elbow partners. The person whose birthday is closest to today will go first. Whose birthday is closest?

Steve raises his hand.

> Steve. Here is our first set. Class, we are placing $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ on the number line. Please draw a number line on your lapboards with your partners. You are to show any benchmarks you need. Steve, you and your group may use any benchmarks I have provided or none at all. Remember, if you need a number that I did not prepare, you may write it on a blank card.

After some rich conversation at the clothesline, students place the cards in the wrong order (see Figure I.4) and return to their seats.

Figure I.4 First Clothesline Placement
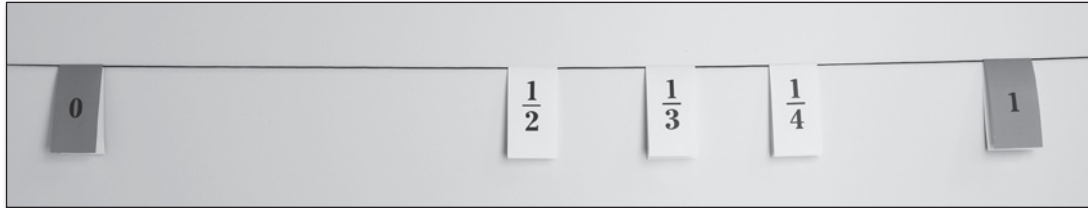
> **Class, let's see who agrees with this response. Please declare your lapboards.**

Student pairs raise their lapboards overhead to display their answers. About half the class agrees with the improper placement of the three values. The other students have them properly placed.

> **Steve, I see that your group used the benchmarks 0 and 1. So, are you claiming that all three fractions are less than 1, but greater than 0?**

Steve: Yes.

Teacher: Why did you place the fractions in this order?

Steve: Because we saw the numbers 2, 3, and 4, so we just counted up.

Teacher: Most of the class agrees with your response here. Does anyone agree with the order of the numbers but for a different reason?

Jackson raises his hand.

> **Yes, Jackson?**

Jackson: I did it in that order because you gave them to us in that order.

Teacher: You disagreed, Kendra. Why?

Kendra: Because $\frac{1}{4}$ is smaller than $\frac{1}{3}$ which is smaller than $\frac{1}{2}$.

Teacher: How do you know?

Kendra: If I cut one pizza into two slices, another pizza into three slices, another into four slices, and then take one slice from each, half of a pizza will be bigger than a fourth of a pizza, and a third will be in the middle.

Teacher: Assuming the pizzas are the same size to begin with?

Kendra: Yes.

Teacher: You have now heard three different explanations. Let's thumb vote: thumbs-up if you agree with the current placement; thumbs-down if you do not. Are these fractions in the correct order?

The whole class votes thumbs-down.

Steve, your group voted against your own response. If you would like to change it, please feel free to come up and do so.

Steve's group returns to the front of the classroom and rearranges the cards (see Figure I.5).

Figure I.5 Second Clothesline Placement
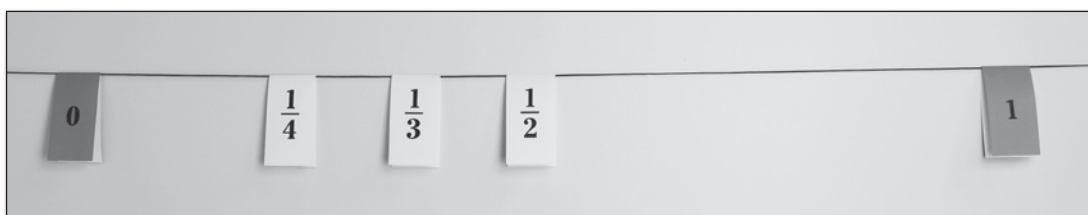
So, the first thing we want to do with the clothesline is *place* the cards. That means we want to get the values in the correct numerical order. We will talk about spacing in a minute, but do you all agree that these three are now in the right order? All in favor, say "aye."

Class: Aye!

Teacher: I noticed that your group equally spaced the numbers this time. Can someone from the group tell me why?

Akira: Well, we realized $\frac{1}{2}$ had to be in the middle. And $\frac{1}{4}$ is half of a half. Then, $\frac{1}{3}$ goes in between those two.

Teacher: Okay. So after we *place*, we want to *space*. That means we want to slide the values on the number line so they are in their proper locations on the number line. Do you agree these values are properly spaced? Thumb vote.

Most, but not all, vote no.
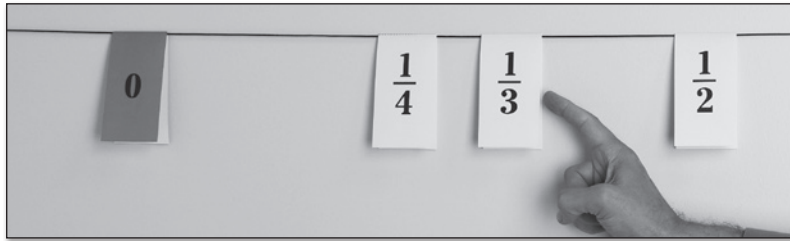
Teacher: Arturo, why not?

Arturo: $\frac{1}{3}$ needs to be closer to $\frac{1}{4}$.

Teacher: Okay, I'm going to tap $\frac{1}{3}$ in the direction in which you point as a class. When the value is in the right place, clap.

Students point to the left as the teacher moves the cards, as shown in Figure I.6. Several are mumbling, "Keep going." There is finally a collective clap.

Figure I.6 Third Clothesline Placement

Who wants to explain why we stopped here? Cindy.

Cindy: I converted the fractions to decimals.

Teacher: How did that help you?

Cindy: You get 0.25, 0.33, and 0.50. I know that 33 is closer to 25 than 50, so I kind of estimated where 0.33 would go.

Teacher: Thank you. Did anyone do it a different way? Skylar.

Skylar: Common denominators give you $\frac{3}{12}$, $\frac{4}{12}$, and $\frac{6}{12}$. The number 4 is closer to 3 than 6. So, I estimated by counting 3, 4, 5, 6.

Teacher: Thank you. Any other ways to do this? Jesse.

Jesse: On our board, I used my pencil to measure $\frac{1}{3}$ and then checked whether that length fit into 1 three times.

Teacher: So, you are using something we call finger reasoning. Everyone, close one eye and place your fingers in front of you so you can remotely measure from 0 to $\frac{1}{3}$.

Students close their eyes and hold up their fingers.

Now, see whether three times that distance is equivalent to 1.

Students use their fingers to remotely measure the spacing of the fractions on the clothesline, as shown in Figure I.7.
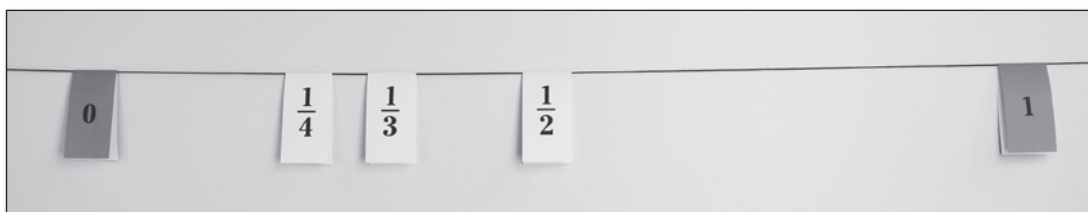
Figure I.7 Sample Finger Measuring
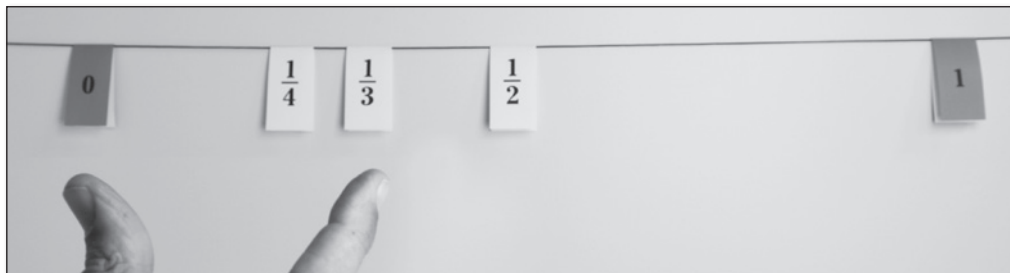
If you now agree with both the placing and the spacing of our three values, clap twice.

The class gives a resounding two claps. The cards stay as they are, as shown in Figure I.8.

Figure I.8 Final Clothesline Placement

Now class, it is time to record our discussions, deductions, and decisions on your activity sheets. On the three blanks, write $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Then, record them on the number line provided with the benchmarks. Be sure to be just as diligent about placing and spacing on your paper as we were on our clothesline. Well done, math crew.

Why These Numbers?

The key to a good clothesline lesson is in the selection of the proper values to pose to the class. For example, I could have chosen any three fractions, but $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ are the most common. In addition, the consecutive denominators of 2, 3, and 4 keep calculations simple and bring out the misconceptions that a larger denominator implies a larger fraction.

The Key Questions

- Why did you include 0 and 1 as benchmarks?
- Why did you not include any benchmarks?
- Do we need benchmarks?
- Why did you place them in that order?
- How do you know $\frac{1}{4}$ is smaller than $\frac{1}{3}$?
- Did anyone come to the same conclusion, but in a different way?
- Why do you disagree?

Analysis

For the record, this example occurred during an Algebra 2 class. A room full of juniors in high school struggled with accurately placing the three simplest fractions on a number line. With these three fractions, I anticipated some students would make the error of placing them in ascending order of the denominator, as some indeed did. I did not expect students to simply copy the order in which I offered the three values. This reveals just how trained students are to copy from and regurgitate back to the teacher.

The strategies of classmates, particularly those of decimal conversions and common denominators, helped many students sort their thinking on fractions. The proportional reasoning encouraged by the finger reasoning was beneficial to all and will be further enhanced as we revisit number sense on the clothesline throughout the school year.

Extension

Place a fraction on the number line that is between $\frac{1}{3}$ and $\frac{1}{2}$.

Figure I.9 Final Clothesline Placement

The Structure of This Book

What you just read was a typical introduction to Clothesline Math.

The majority of this book is filled with lessons like these. The values or expressions to be placed on the number line are posed first, followed by a mock classroom discourse that leads to the correct placement on the clothesline, which is to be recorded on the *Clothesline Math* activity sheet (pages 315–316) at the end of each lesson. A teacher debrief finishes each model lesson with the following format:

Why These Numbers: This is an explanation of the importance behind the choice of numbers for each lesson.

Key Questions: Clothesline Math is intended for discourse, not lecture. Since good classroom discussion is led by questions, not statements, a bank of prompts is offered for each lesson.

Analysis: This is the reflection on the lesson vignette, including discussion of student thinking and teacher decisions during the lesson.

Extension: This is an additional activity suggestion to be used for differentiation, assessment, or independent assignment.
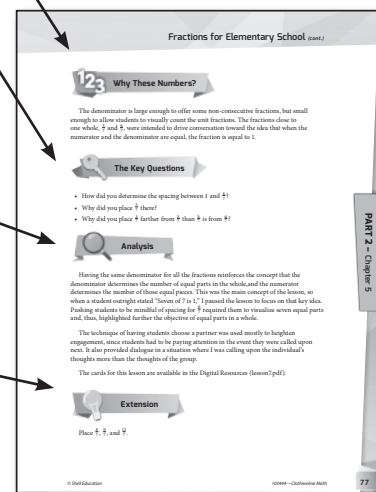
This text also includes justification for teaching number sense at all grade levels, the simple materials and setup needed to conduct clothesline lessons in your classroom, and techniques for effectively facilitating the discourse spawned by our favorite number line tool. With these examples and tips, I hope to empower your teaching with Clothesline Math.