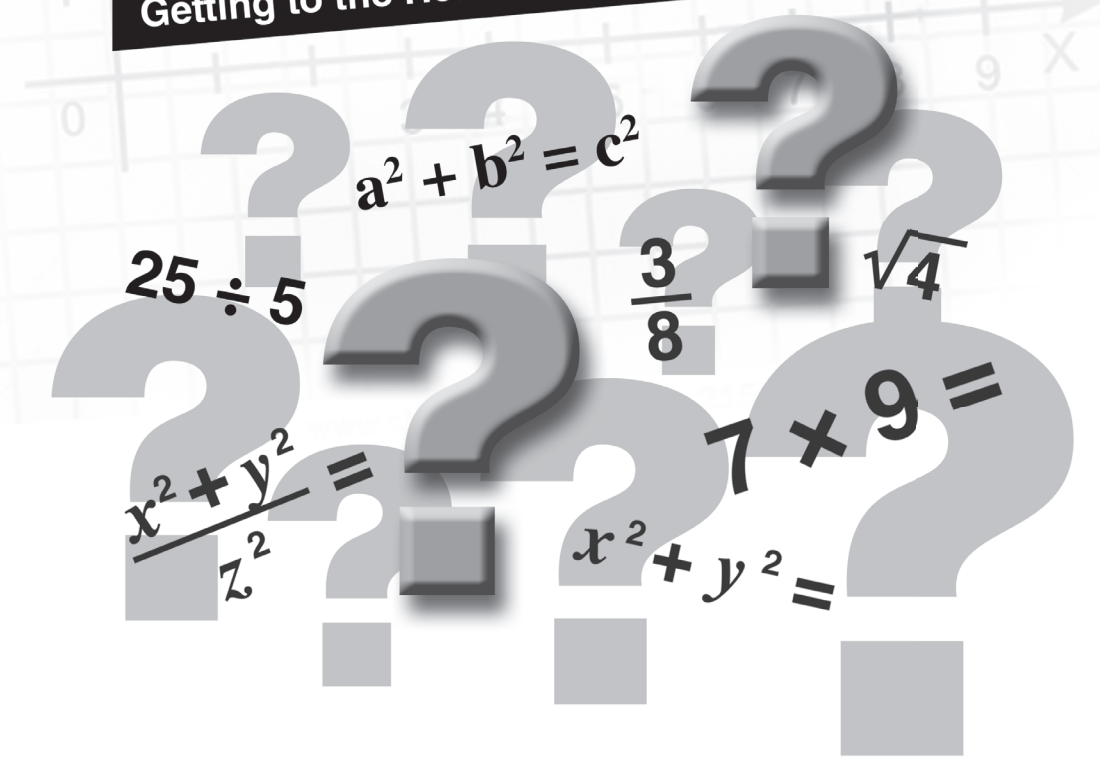
WHAT'S MATH YOUR PROBLEM!?!

Getting to the Heart of Teaching Problem Solving



Author
Linda Gojak

Foreword
Laney Sammons

SHELL EDUCATION

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Introduction

The purpose of this book is to help you help students be successful mathematical problem solvers. To do this, we must become more comfortable with problem solving ourselves. We need to examine our own beliefs about problem solving, how we can more effectively incorporate rich problems into our mathematics instruction, and we need to solve problems! This book presents a strategy-based approach to problem solving and models how to use the strategies effectively with students. The following strategies are discussed throughout the book:

- Restate the Problem in Your Own Words
- Identify Wanted, Given, and Needed Information
- Identify a Subgoal
- Select Appropriate Notation
- Look for a Pattern
- Create a Table
- Create an Organized List
- Guess and Check
- Make a Model
- Draw a Picture or Diagram
- Act It Out
- Create or Use a Graph
- Solve a Simpler Problem
- Account for All Possibilities
- Work Backwards
- Change Your Point of View

Chapter 1 addresses general beliefs about problem solving for students in kindergarten through grade 8. It helps present the "5 Ws and an H" behind problem solving.

Management and organizational strategies and suggestions to successfully implement problem solving into mathematics instruction appear in **Chapter 2**, along with ideas for differentiation and grouping students to best meet their needs.

Chapters 3 through 6 present an extensive—but not exhaustive—list of problem-solving strategies. Each section begins with a description of the strategy and offers suggestions for its introduction to students, followed by several sample problems. Problems are assigned a grade-level range to give you an idea of the type of task to use to help students become comfortable using the strategy. Also included are rich mathematical problems that can be solved using the strategies suggested. Work these problems in your problem-solving journal. You will become a better problem solver by solving problems! In addition you will grow professionally and become familiar with how to use each strategy. Here are a few suggestions for ways to work these problems:

- **Alone:** Spend as much time as you need. The solutions shown are only suggestions. Rich problems often have several solution methods.
- **As part of a professional study group:** Work your way through the book with your colleagues. Solve the problems and discuss your thinking. Or, assign problems to your students and use the study group to review and discuss student work.
- **As professional development:** The strategies and problems in this book can be used in a class or professional development workshop.

Although suggestions are included with the strategies, explanations, and solutions, the best way you can learn and grow from this work is to make it your own. Compare your solution process with mine. Compare your students' work with your work.

In Chapter 7, assessment is addressed with suggestions for how to informally and formally assess students' mathematical reasoning and understanding.

Chapter 8 includes answers to the most frequently asked questions about problem solving from teachers around the world that I have met over the years in professional development settings.

Reflection questions are included in every chapter to encourage you to think about your own beliefs about problem solving. Include your reflections in your problem-solving journal as you journey through this book and answer them as you read. As you implement some of these ideas with your students, reread your reflections. Are you thinking differently about any of these ideas, or are your beliefs reinforced? Most of all, enjoy your problem-solving journal. Use drawings, markers, tables, and charts! Organize it in a way that is most useful to you. Refer back to it often and extend it beyond the problems in this book. As you continue on the road to professional growth, you will find new ideas and more problems to solve. And, don't be surprised if some of the best ideas come from your students!

Happy problem solving!

—Linda Gojak

Choosing a Good Problem to Use With Your Students

- Consider the mathematics you want students to learn.
- Find or modify a problem that will be of interest to the students.
- Consider the difficulty level of the problem.
- Consider how you will introduce the problem to the class:
  - To make it understandable.
  - To make it interesting and motivating.
- Consider what is involved in the solution process and how much time it will take students to solve it.

A Problem-Solving Framework

George Polya, a 20th century mathematician, was a great advocate for the use of problem-solving techniques in learning mathematics. His most famous book, *How To Solve It*, was first published in 1945. Since then it has sold over a million copies and is still in publication today.

In *How To Solve It*, Polya identifies four principles of problem solving. These principles provide a framework to help students approach any type of problem. Interestingly, Polya's principles can also be applied to problems outside of mathematics and are often applied in doing science-related activities.

Principle One: Understand the Problem

Too often students jump into solving a problem without considering all of the details and any special situations. What does understanding a problem entail? How do you know when you understand the problem? More importantly, how do we ensure that a student knows when he understands the problem he is trying to solve?

This principle includes several of the problem-solving strategies from the later chapters of this book. They are as follows:

- Restate the Problem in Your Own Words
- Identify Wanted, Given, and Needed Information
- Identify a Subgoal
- Draw a Picture or Diagram
- Select Appropriate Notation

Although there is no particular order to the strategies used to recognize all of the essential details in a problem, each is an important step in comprehending the problem. Students should begin by reading the problem and restating it in their own words. Visualizing the problem will help to focus on the important details. Make students aware that it often takes more than one reading to identify the critical details. Identifying the wanted, given, and needed information includes focusing on the question asked to begin to consider how to get started and what to do. What data do I have? What is unknown? What am I looking for to reach a solution? Students also must understand whether the problem involves multiple steps and if a subgoal must be identified. Drawing a picture or diagram can add to the understanding of what is given and, at the same time, can help to focus on what is needed to solve the problem. Also consider the type of notation to use, which may be modeling with concrete materials to using mathematical symbols that will be helpful in getting started on the problem.

Stop and Think

Respond to the questions below in your problem-solving journal.

- Choose a rich problem that you have used with students in the past or that you would like to use with them. Think about how you developed an understanding of the problem.
- What steps did you take to make sense of the situation to help you get started?

Principle Two: Devise a Plan

Now that you have a clearer understanding of the problem situation, it is time to consider how you will go about solving it. It may be simple calculations that will enable you to reach a solution. For many students, determining which mathematical operation to use presents a challenge. This is often because they have not associated meaning with the facts of the problem and the question being asked, or they do not have a deep understanding of the meaning of the operation and depend on "key words" (which don't work!) to tell them what to do.

Rigorous problems often involve more than one or two computational steps. You have to determine how to get started, what approach may lead to a successful solution, and how you will represent your thinking in order to carry out your plan. This is where the problem-solving strategies come in. It is a good idea to have a list of the strategies available for students to consider every time they solve a problem. Think about which of the strategies may be helpful. Too often, students do not take the time to think about appropriate strategies before they begin to attempt a solution.

Once you have determined which strategies will help you to get started, think about how you will relate the information in the problem to using the strategies. If you decide that making a table will help you use the information in the problem, you need to consider what information should be used in the table. If you choose to solve a simpler problem first, how will you modify the problem so you can get started? How will you use the information to determine if there is a pattern you can complete to reach the original solution?

Stop and Think

Respond to the questions below in your problem-solving journal.

Look back at the problem you selected on page 44.

- What strategies did you decide to use?
- How did you choose those strategies?

Principle Three: Carry out the Plan

Once you have decided which strategies to use, it is time to implement the plan. An important part of this principle is to check each step as you proceed. Asking yourself questions throughout the solution process will help you to know if you are headed in a direction that is leading towards an acceptable solution or if you must change your course. Sometimes you will get started and find you need to change your approach by adjusting a strategy, adding more strategies, or selecting an entirely different approach. This is all part of learning to be a problem solver. Although this can be frustrating for students, it is also what provides the satisfaction of reaching the solution and understanding the steps and effort it took to get there!